



Pearson



# Mark Scheme (Results)

Summer 2017

Pearson Edexcel GCE Mathematics/Further  
Mathematics

Decision Mathematics D1 (6689/01)

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

## EDEXCEL GCE MATHEMATICS

### General Instructions for Marking

1. The total number of marks for the paper is 75.
2. The Edexcel Mathematics mark schemes use the following types of marks:
  - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
  - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
  - **B** marks are unconditional accuracy marks (independent of M marks)
  - Marks should not be subdivided.
3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol  $\checkmark$  will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- \* The answer is printed on the paper
- $\square$  The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
6. If a candidate makes more than one attempt at any question:
  - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
  - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
7. Ignore wrong working or incorrect statements following a correct answer.

Question Number	Scheme	Marks
1. (a)(i)	A bipartite graph consists of two sets of vertices X and Y	B1
	The edges only join vertices in X to vertices in Y, not vertices within a set	B1
(a)(ii)	A path from an unmatched vertex in one set to an unmatched vertex in the other set which alternately uses arcs not in/in the matching	B1
		B1 (4)
<b>SC: APs FROM F TO 2 AND FROM C TO EITHER 2 OR 5 - SEE NOTES</b>		
(b)	Alternating path: $F - 1 = A - 3 = B - 4 = D - 6 = E - 5$	M1
	Change status: $F = 1 - A = 3 - B = 4 - D = 6 - E = 5$	A1
	Improved matching: $A = 3, B = 4, (C \text{ unmatched}), D = 6, E = 5, F = 1$	A1 (3)
(c)	Alternating path: $C - 5 = E - 6 = D - 4 = B - 2$	M1
	Change status: $C = 5 - E = 6 - D = 4 - B = 2$	A1
	Complete matching: $A = 3, B = 2, C = 5, D = 4, E = 6, F = 1$	A1 (3)
		<b>10 marks</b>

### Notes for Question 1

**ai1B1: Two sets of vertices** – must contain the three words in bold – accept nodes for vertices but not points or any other non-technical language

**ai2B1:** (Edges) must go from one (set) into the other – candidates must give an indication of going from one set to the other – however, they do not need to use the word ‘set’ for this mark. Candidates do not need to mention that edges should not join vertices within a set but if a candidate does imply that a bipartite graph can join vertices within a set then withhold this mark (**no isw**). If a candidate **only** says that you cannot connect nodes from the same set then this is B0. As an absolute minimum accept a statement along the lines of: ‘must go from one to the other’ – note that for this mark technical language may be absent or incorrect.

**aii3B1:** unmatched to unmatched (vertices do not need to be explicitly mentioned for this mark but B0 if arcs implied)

**aii4B1:** (alternate) **arcs** not in/in (not vertices/nodes) – **must** mention arcs/edges (not lines) and an understanding of what ‘alternating’ means in this context

**b1M1:** An alternating path (e.g. letter 1<sup>st</sup> set – number 2<sup>nd</sup> set – letter 1<sup>st</sup> set – ...) from F to 5 or vice-versa

**b1A1:** CAO – a correct path including change status **either** stated (only accept ‘change (of) status’ **or** ‘c.s’ but not, e.g. ‘change state’) **or** shown (all symbols e.g. (... – ... = ... – ...) interchanged (... = ... – ... = ...)) Chosen path clear

e.g.

- $F * 1 = A * 3 = B * 4 = D * 6 = E * 5$   
 $F = 1 * A = 3 * B = 4 * D = 6 * E = 5$ 
scores M1A1 (change status shown)
- change status  $F - 1 = A - 3 = B - 4 = D - 6 = E - 5$ 
scores M1A1 (change status stated)
- c.s.  $F - 1 = A - 3 = B - 4 = D - 6 = E - 5$ 
scores M1A1 (change status stated)
- $F - 1 = A - 3 = B - 4 = D - 6 = E - 5$   
 c.s.  $F = 1 - A = 3 - B = 4 - D = 6 - E = 5$ 
scores M1A1 (change status stated and shown)
- $F - 1 = A - 3 = B - 4 = D - 6 = E - 5$   
 $F = 1, A = 3, B = 4, \dots$ 
scores M1A0 (no change status stated or shown)

Question Number	Scheme	Marks
	<p><b>b2A1:</b> CAO – improved matching - must follow from the correct stated path. Accept either stated <b>or</b> on a clear diagram (with five arcs <b>only</b>). <b>Please check the top of the second page as many candidates will draw either the improved or complete matching on the nodes provided there</b></p> <p><b>c1M1:</b> A second alternating path from C to 2 or vice-versa  <b>c1A1:</b> CAO – a correct path <b>including</b> change status stated <b>or</b> shown. Chosen path clear  <b>c2A1:</b> CAO (complete matching) must follow from two correct stated paths (so <b>both</b> previous M marks must have been awarded). Accept on a clear diagram (with six arcs <b>only</b>)</p> <p><b>Special Cases for (b) and (c):</b></p> <p><b>Alternating path from F to 2</b></p> <p>Candidates who find an alternating path from F to 2 can score in (b)</p> <p>M1 for an alternating path from F to 2 (or vice-versa),  A1 for the correct alternating path (<math>F - 1 = A - 3 = B - 2</math>) <b>and</b> change of status (stated or shown)  A1 for the correct improved matching of <math>A = 3, B = 2, D = 4, E = 6, F = 1</math> from the correct stated path</p> <p>In (c) the alternating path is simply <math>C - 5</math> and therefore no marks in (c) – so an alternating path from F to 2 can score a maximum of three marks (of the six available) in (b) and (c)</p> <p><b>Alternating path from either C to 2 or C to 5</b></p> <p>Candidates who find in (b) an alternating path from either C to 2 <b>or</b> C to 5 can score in (b)</p> <p>M1 for an alternating path from either C to 2 <b>or</b> C to 5  A1 for either <math>C - 3 = B - 2</math> <b>or</b> <math>C - 3 = B - 4 = D - 6 = E - 5</math> together with the change of status (either stated or shown)  A0</p> <p>In (c)</p> <p>M1 for either <math>F - 1 = A - 3 = C - 5</math> (following their path from C to 2) <b>or</b> <math>F - 1 = A - 3 = C - 5 = E - 6 = D - 4 = B - 2</math> (following their path from C to 5)  A0  A0</p> <p>So both special cases can score a maximum of three marks (of the six available in (b) and (c))</p>	

Question Number	Scheme	Marks
2. (a)	BC, BE, EH, CD; EF, FG; HJ, AG	M1; A1; A1 (3)
(b)	(£) 380	B1 (1)
(c)	BC, EH, BE, reject CD, EF, reject BF, FG, reject HJ, AG (reject BG), (reject DA), (reject BA) (Note BD and FJ are already in the tree)	M1 A1 A1 (3)
		<b>7 marks</b>

**Notes for Question 2**

**a1M1:** First four arcs correctly chosen in order (BC, BE, EH, CD) or first five nodes correctly chosen in order (B, C, E, H, D). **If any explicit rejections seen at any point then M1 (max) only.** Allow a list of weights for M1 **only** (10, 30, 25, 45). Candidates may apply Prim's in matrix form so the order of the nodes may be seen at the top of a matrix – accept {-, 1, 2, 5, 3, -, -, 4, -} for the M mark. Allow CB for BC etc. throughout (a) and (c)

**a1A1:** First six arcs correctly chosen in order (BC, BE, EH, CD, EF, FG) **or** all nine nodes correctly chosen in order (B, C, E, H, D, F, G, J, A). Candidates may apply Prim's in matrix form so the order of the nodes may be seen at the top of a matrix – accept {9, 1, 2, 5, 3, 6, 7, 4, 8} – do not condone any missing numbers e.g. the number 9 must be above A

**a2A1:** CSO (correct solution only) – all arcs correctly stated and chosen in the correct order. Candidates must be considering arcs for this final mark (do not accept a list of nodes or numbers across the top of the matrix unless the correct list of arcs (in the correct order) is also seen)

**Misread:** Starting at a node other than B scores M1 only in (a) – must have the first four arcs (or five nodes) correct (and in the correct order). The most common misread is those that start at A so for M1 only – accept AG, GF, EF, EH or A, G, F, E, H or 75, 70, 52, 25

**b1B1:** CAO (condone lack of £)

**c1M1:** Kruskal's - first three arcs BC, EH, BE,...(or weights 10, 25, 30,...) chosen correctly in order **and at least one rejection seen at some point.** Ignore any reference to BD and FJ

**c1A1:** All six arcs BC, EH, BE, EF, FG, AG chosen correctly in order and no additional arcs. Ignore any reference to BD and FJ. Do not accept only a list of weights for this mark

**c2A1:** CAO all selections and rejections correct in the correct order and at the correct time. Ignore any reference to BD and FJ. Do not accept only a list of weights for this mark – do not need to see explicit rejection of BG, DA and/or BA for this mark

- Listing all the arcs in order and then listing those arcs in the tree in the correct order is fine for full marks as this implies that rejections are correct and at the correct time
- Listing all the arcs in order and just drawing the MST is M0

No misread if candidates use any other arcs instead of BD and/or FJ



Question Number	Scheme	Marks
3. (a)	Bin 1: $\boxed{42}$ $\boxed{21}$ Bin 2: $\boxed{15}$ $\boxed{16}$ 10 11 Bin 3: $\boxed{35}$ 27 Bin 4: $\boxed{31}$ Bin 5: 39	$\boxed{M1}$ $\underline{A1}$ A1 (3)
(b)	e.g. middle right 42 21 15 16 35 <u>10</u> 31 11 27 39 Pivot 10 42 21 15 16 <u>35</u> 31 11 27 39 $\boxed{10}$ Pivot 35 42 <u>39</u> $\boxed{35}$ 21 15 16 <u>31</u> 11 27 $\boxed{10}$ Pivot 39, 31 42 $\boxed{39}$ $\boxed{35}$ $\boxed{31}$ 21 15 <u>16</u> 11 27 $\boxed{10}$ Pivots (42) 16 42 $\boxed{39}$ $\boxed{35}$ $\boxed{31}$ 21 <u>27</u> $\boxed{16}$ 15 <u>11</u> $\boxed{10}$ Pivots 27, 11 42 $\boxed{39}$ $\boxed{35}$ $\boxed{31}$ $\boxed{27}$ 21 $\boxed{16}$ 15 $\boxed{11}$ $\boxed{10}$ Sort complete	M1 A1 A1ft A1 (4)
(c)	Bin 1: $\boxed{42}$ $\boxed{21}$ Bin 2: $\boxed{39}$ $\boxed{16}$ 10 Bin 3: $\boxed{35}$ $\boxed{27}$ Bin 4: $\boxed{31}$ $\boxed{15}$ 11	$\boxed{M1}$ $\underline{A1}$ A1 (3)
(d)	$14 < x < 17$	B3,2,1,0 (3)
		<b>13 marks</b>

### Notes for Question 3

**a1M1:** First five items placed correctly and at least eight values placed in bins - condone cumulative totals for M1 only (the boxed values)

**a1A1:** First eight items placed correctly (the boxed **and** underlined values)

**a2A1:** CSO (so no additional/repeated values)

**b1M1:** Quick sort, pivot, p, chosen (must be choosing middle left or right – choosing first/last item as the pivot is M0). After the first pass the list must read (values greater than the pivot), pivot, (values less than the pivot). **If only choosing one pivot per iteration then M1 only** – Bubble sort is not a MR and scores M1 only for 42 21 16 35 15 31 11 27 39 10 (for left to right) **or** 42 39 21 15 16 35 10 31 11 27 (for right to left)

**b1A1:** First two passes correct **and** next pivots chosen correctly for the third pass (but the third pass does not need to be correct) – so they must be choosing (if middle right) pivot values of 39 and 31 for the third pass or (if middle left) a pivot value of 16

**b2A1ft:** Third and fourth passes correct (follow through from their second pass and choice of pivots). They do not need to be choosing a pivot for the fifth pass for this mark

**b3A1:** CSO (correct solution only – all previous marks in this part **must** have been awarded) including a fifth pass in which the 27 and 11 (if middle right) or 21 (if middle left) is used as a pivot and a ‘sort complete’ - this could be shown by the final list being re-written or ‘sorted’ statement or each item being used as a pivot (which would therefore mean that the final list would have been written twice)

**c1M1: Must be using ‘sorted’ list** in descending order. First five items placed correctly and at least eight values placed in bins – condone cumulative totals for M1 only (the boxed values)

**c1A1:** First eight items placed correctly (the underlined **and** boxed values)

**c2A1:** CSO (so no additional/repeated values)

**SC for part (c)** – if ‘sorted’ list is incorrect from part (b) and M0 would be awarded in (c) then award M1 only in (c) for their first eight items correctly placed – by ‘incorrect’ they can have only one ‘error’ - an ‘error’ is one missing number, one extra number, or one number incorrectly placed. Allow full marks in (c) if a correct list is used in (c) even if the list is incorrect at the end of (b)

Question Number	Scheme	Marks
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**IN (d) mark their final attempt only – if in doubt please send to review**

**d1B1:** Any inequality in  $x$  linked with either 14 **or** 17 (allow in words or using set notation) **or** an indication of an interval with 14 **and** 17 but with no inequality sign (or even  $x$ ) present

**d2B1:** Any inequality in  $x$  linked with 14 **and** 17 (allow in words) or using set notation (14, 17)

**d3B1:** CAO (allow as two separate inequalities:  $x > 14, x < 17$  (allow without a comma separating the two inequalities), accept  $x > 14$  **and**  $x < 17$  but not  $x > 14$  **or**  $x < 17$ ) - if defined using words only then do not award this mark

**SC: award B1B1B0 in (d) for  $15 \leq x \leq 16$  or [15,16] (no marks for any strict inequalities with 15/16)**

**Misreads**

- If the candidate has misread a number at the start of (a), so genuinely miscopy a number then mark the whole of (a), (b) and (c) as a misread – removing the last two A marks earned. This gives a maximum of 8 marks in total for these three parts
- If they have used the correct numbers at any point in (a) and then use an incorrect number in (b) (say 24 instead of 42) from the beginning of the sort or misread one of their own numbers during (b) then count it as one ‘error’ in (b) (so they will lose at least the final A mark in (b) but should be able to gain at least the M mark and the follow through A mark) – then mark (c) according to the SC above. More than one ‘error’ in (b) loses all subsequent A marks in (b)

**Sorting list into ascending order in (b)**

- If the candidate sorts the list into ascending order and reverses the list in this part then this can score full marks in (b)
- If the list is not reversed in (b) then mark as a misread (so remove the last two A marks earned in (b)). If the list is reversed at the start of (c) but not in (b) then still treat this as a misread. If the list is in ascending order in (b) award no marks for first-fit increasing in (c). If the candidate says that the list needs reversing in (b) but does not actually show the reversed list in (b) then remove the final A mark

**Middle left for (b):**

42	21	15	16	<u>35</u>	10	31	11	27	39	Pivot 35
<u>42</u>	39	<u>35</u>	21	15	16	<u>10</u>	31	11	27	Pivots 42, 10
<u>42</u>	39	<u>35</u>	21	15	<u>16</u>	31	11	27	<u>10</u>	Pivots (39) 16
<u>42</u>	39	<u>35</u>	21	<u>31</u>	27	<u>16</u>	<u>15</u>	11	<u>10</u>	Pivots 31, 15
<u>42</u>	39	<u>35</u>	<u>31</u>	<u>21</u>	27	<u>16</u>	<u>15</u>	11	<u>10</u>	Pivot 21 (11)
<u>42</u>	39	<u>35</u>	<u>31</u>	27	<u>21</u>	<u>16</u>	<u>15</u>	11	<u>10</u>	Sort complete

**Ascending middle right**

42	21	15	16	35	<u>10</u>	31	11	27	39
<u>10</u>	42	21	15	16	<u>35</u>	31	11	27	39
<u>10</u>	21	15	16	<u>31</u>	11	27	<u>35</u>	42	<u>39</u>
<u>10</u>	21	15	<u>16</u>	11	27	<u>31</u>	<u>35</u>	<u>39</u>	42
<u>10</u>	15	<u>11</u>	<u>16</u>	21	<u>27</u>	<u>31</u>	<u>35</u>	<u>39</u>	42
<u>10</u>	<u>11</u>	15	<u>16</u>	21	<u>27</u>	<u>31</u>	<u>35</u>	<u>39</u>	42

**Ascending middle left**

42	21	15	16	<u>35</u>	10	31	11	27	39
21	15	16	<u>10</u>	31	11	27	<u>35</u>	<u>42</u>	39
<u>10</u>	21	15	<u>16</u>	31	11	27	<u>35</u>	39	<u>42</u>
<u>10</u>	<u>15</u>	11	<u>16</u>	21	<u>31</u>	27	<u>35</u>	39	<u>42</u>
<u>10</u>	11	<u>15</u>	<u>16</u>	<u>21</u>	27	<u>31</u>	<u>35</u>	39	<u>42</u>
<u>10</u>	11	<u>15</u>	<u>16</u>	<u>21</u>	27	<u>31</u>	<u>35</u>	39	<u>42</u>

(Sort complete)

Question Number	Scheme	Marks
4. (a)		M1 A1 (DCBE) A1 (FG) A1ft (JH)
	Shortest path A to H: ADCBEFJH	A1
	Length of shortest path: 20 (miles)	A1ft (6)
(b)	$AD + E(F)G = 5 + 5 = 10^*$ $A(DCB)E + D(CBEF)G = 12 + 12 = 24$ $A(DCBEF)G + D(CB)E = 17 + 7 = 24$	M1 A1 A1
	Repeat edges: AD, EF, FG	A1
	Length = $85 + 10 = 95$ (miles)	A1ft (5)
(c)(i)	AD, CD, BC, BG	B1
(ii)	Route: e.g. ADABCDCBFEHJFGBGA	B1
	Length = $85 + 18 - 6 - 2 = 95$ (miles)	M1 A1 (4)
		15 marks

**Notes for Question 4**

**In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at G the working values must be 20 18 17 in that order (20 17 18 is incorrect) and with no additional working values. It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, ... will be penalised once (see notes below) but 1, 2, 3, 5, 6, ... is fine. Errors in the final values and working values are penalised before errors in the order of labelling**

**a1M1:** A larger value replaced by a smaller value at least once in the working values at either B or F or G or H

**a1A1:** All values at D, C, B and E correct and the working values in the correct order at B. Condone lack of 0 in A's working value – please check carefully for a 5 in the working values at D

**a2A1:** All values at F and G correct and the working values in the correct order. Penalise order of labelling only once per question (F and G must be labelled in that order and F labelled after A, D, C, B and E)

Question Number	Scheme	Marks
	<p><b>a3A1ft:</b> All values in J and H correct on the follow through and the working values in the correct order. Penalise order of labelling only once per question (J and H must be labelled in that order and J labelled after all other nodes (excluding H)). To follow through J check that the working value at J follows from the candidate's final value from F and that the final value, and order of labelling, follows through correctly. Repeat this process for H (which will have working values from E and J)</p> <p><b>a4A1:</b> CAO for the path (from either A to H or H to A)</p> <p><b>a5A1ft:</b> If their answer is not 20 follow through their final value at H (ignore units)</p> <p><b>b1M1:</b> Three distinct pairings of A, D, E and G</p> <p><b>b1A1:</b> Any two rows correct including pairings <b>and</b> totals</p> <p><b>b2A1:</b> All three rows correct including pairings <b>and</b> totals</p> <p><b>b3A1:</b> CAO correct <b>edges</b> clearly (not just in their working) stated as AD, EF, FG. Do not accept EG, EFG or or EG via F</p> <p><b>b4A1ft:</b> 85 + their smallest repeat out of a choice of at least <b>two</b> totals seen (the correct answer of 95 must come from the correct pairing of AD and EG)</p> <p><b>c1B1:</b> CAO (AD, CD, BC, BG)</p> <p><b>c2B1:</b> Any correct route (the route may be given in terms of either vertices (ADA...) or arcs (AD, DA, AB,...)) – checks: start and finish at A, 17 vertices (repeats AD, CD, BC, BG, and nodes A, B appear three times, C, D, F, G appear twice and E, H and J appear once)</p> <p><b>c1M1:</b> <math>85 - 6 - 2 + '18'</math> (where '18' is the sum of their repeated arcs from A to G in c1B1 (this may need to be checked) or a clear attempt at their shortest path from A to G) <b>or</b> <math>85 - 8 + '18'</math> <b>or</b> 95 <b>or</b> for adding the weights of all the arcs from their route (so ft their answers from c1B1 and c2B1)</p> <p><b>c1A1:</b> CAO of 95 including as a minimum the working <math>85 + 18 - 8</math> or explicitly adding the weights of <b>all</b> the arcs from a correct route given in c2B1. Note that <math>85 + 10</math> or <math>77 + 18</math> alone is not sufficient for this mark (ignore units)</p>	

Question Number	Scheme	Marks
<b>5. (a)</b>	$2y \geq x$ $5y + 2x \leq 50$ $2x + y \geq 10$	B2, 1, 0 (2)
<b>(b)</b>	(4,2), (0,10)	B1
	$\left(\frac{100}{9}, \frac{50}{9}\right)$ or $\left(11\frac{1}{9}, 5\frac{5}{9}\right)$	M1 A1 (3)
<b>(c)</b>	$(0,10) \rightarrow P = 30$ $(4,2) \rightarrow P = 14$ $\left(\frac{100}{9}, \frac{50}{9}\right) \rightarrow P = \frac{350}{9}$ or $38\frac{8}{9}$ so optimal vertex is $\left(\frac{100}{9}, \frac{50}{9}\right)$	M1 A1 (2)
<b>(d)</b>	$Q = 2x + \lambda y$ $2\left(\frac{100}{9}\right) + \lambda\left(\frac{50}{9}\right) > 2(0) + \lambda(10)$ or objective line method (see Way 2) $\Rightarrow \lambda < 5$ $2\left(\frac{100}{9}\right) + \lambda\left(\frac{50}{9}\right) > 2(4) + \lambda(2)$ or objective line method (see Way 2) $\Rightarrow \lambda > -4$ $(-4 < \lambda < 5)$	M1 A1 M1 A1 <b>(4)</b>
		<b>11 marks</b>

**Notes for Question 5**

**a1B1:** Any two correct (accept strict inequalities) – accept equivalent inequalities

**a2B1:** CAO (accept equivalent inequalities)

**b1B1:** CAO for both integer coordinates – accept  $x = 4, y = 2$ , etc.

**b1M1:** Using simultaneous equations to find the non-integer vertex – must get to  $x = \dots$  and  $y = \dots$  Must be a correct method to solve simultaneous equations but allow slips/errors. If no working present then this mark can be awarded for an awrt (11.1, 5.56) or (11.1, 5.55)

**b1A1:** CAO – must be exact (condone correct recurring decimal notation). If correct answer seen with no working then award M1 A1 in this part. ISW if correct exact answer seen which is then given in non-exact form

**c1M1:** Testing all three of their vertices in the correct objective function

**c1A1:** Correct **three** values for  $P$  (accept awrt 38.9 for 350/9) and correct optimal vertex either stated or clearly indicated (allow awrt (11.1, 5.56) or (11.1, 5.55))

Question Number	Scheme	Marks
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**d1M1: WAY 1 – point testing:** Their attempt at  $\left(\frac{100}{9}, \frac{50}{9}\right)$  evaluated in  $Q$  compared to  $(0, 10)$  evaluated in

$Q$  - allow any inequality sign or equals

**d1A1:**  $\lambda < 5$  (CAO so allow equals used throughout and then the correct inequality at the end but A0 if incorrect inequality seen in working or if non-exact values used in working)

**d2M1:** Their attempt at  $\left(\frac{100}{9}, \frac{50}{9}\right)$  evaluated in  $Q$  compared to  $(4, 2)$  evaluated in  $Q$  – allow any inequality sign or equals

**d2A1:**  $\lambda > -4$  (CAO - see d1A1). Do not award this mark if candidates give both correct answers and then give an answer of  $0 < \lambda < 4$  or if any additional answers seen

**SC for Way 1:** If optimal vertex in (c) is either  $(4, 2)$  or  $(0, 10)$  then the M mark not awarded in (d) can be awarded for evaluating and comparing  $(4, 2)$  with  $(0, 10)$  in  $Q$ . Therefore an incorrect optimal vertex in (c) can earn at most M1A0M1A0 in (d)

**NOTE that in WAY 2 the 2<sup>nd</sup> A mark is dependent on the correct three vertices of the feasible region**

**d1M1: WAY 2 – objective line:**  $-\frac{2}{\lambda}$  compared to either  $-\frac{2}{5}$  or  $\frac{1}{2}$  or  $-2$  (oe e.g.  $\frac{2}{\lambda}$  with  $\frac{2}{5}$ ,  $\frac{\lambda}{2}$  with  $\frac{5}{2}$ , etc.) so correctly comparing the gradient of the new objective line with any one of the three line segments that define  $R$  – accept any inequality or equals

**d1A1:**  $\lambda < 5$  or  $\lambda > -4$  (CAO so allow equals used throughout and then the correct inequality at the end but A0 if incorrect inequality seen in working)

**d2M1:**  $-\frac{2}{\lambda}$  compared to **both**  $-\frac{2}{5}$  **and**  $\frac{1}{2}$  (oe) – so correctly comparing the gradient of the new objective with the correct two line segments that give the correct optimal vertex - accept any inequality or equals

**d2A1:**  $\lambda < 5$  **and**  $\lambda > -4$  only (CAO - see d1A1) – note that this mark is dependent on all correct three vertices that define the feasible region and must come from correct comparisons with  $-\frac{2}{5}$  **and**  $\frac{1}{2}$ . Do not award this mark if candidates give the correct answer and then give an answer of  $0 < \lambda < 4$  or if any additional answers seen

Note that the correct answers in Way 2 must come from  $-\frac{2}{\lambda} < -\frac{2}{5}$  and  $-\frac{2}{\lambda} > \frac{1}{2}$

**Correct answers with no working**

**Award d1M1 and d1A1 (first two marks) for one correct answer then d2M1 and d2A1 (full marks) for both correct answers only (so no additional answers) – if any of the three vertices of the feasible region are incorrect then award the first three marks for both correct answers only**

Question Number	Scheme	Marks
6. (a)	<p>The network diagram shows activities A through N. Each activity is represented by a box with its duration and ES/EF/LS/LF values. The critical path is A-B-E-I-J-N.</p>	<p>M1 A1 M1 A1 (4)</p>
(b)	<p>The Gantt chart shows the timing of activities A through N. The x-axis represents time from 0 to 28. Activities are shown as horizontal bars with their start and end times. Shaded areas indicate float time.</p>	<p>M1 A1 A1 A1 (4)</p>
(c)	(Activities) J and L (must be happening at time 18.5)	B1 (1)
(d)	The earliest that P can start is at time 9 (the time that the critical activity D must be finished by) – if the duration of P is 10 then the earliest P can finish is $19 < 24$ and so the second project can be completed on time. If the duration of P is 17 days then the earliest P can finish is $26 > 24$ and so the second project cannot be completed on time	B1 B1 (2)
		11 marks

Question Number	Scheme	Marks
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**Notes on Question 6**

**a1M1:** All top boxes complete, values in the top boxes generally increasing in the direction of the arrows ('left to right'), condone one 'rogue' value (if values do not increase in the direction of the arrows then if one value is ignored and then the values do increase in the direction of the arrows then this is considered to be only one rogue value)

**a1A1:** CAO for the top boxes

**a2M1:** All bottom boxes complete, values generally decreasing in the opposite direction of the arrows ('right to left'), condone one rogue. Condone missing 0 and/or 24 for the M only

**a2A1:** CAO for the bottom boxes

**Note that it is acceptable for the critical activities to appear on separate lines or for several activities to appear on the same line as long as their length and floats are clear and do not overlap**

**b1M1:** At least ten activities including at least five floats. A scheduling diagram scores M0

**b1A1:** The critical activities dealt with correctly and appearing just once (C, D, G, I and J) and three non-critical activities dealt with correctly

**b2A1:** Any six non-critical activities correct (this mark is not dependent on the previous A mark)

**b3A1:** CSO – completely correct Gantt chart (exactly fourteen activities appearing just once – ignore any inclusion of activity P)

**c1B1:** CAO (with no additional activities)

**Both marks in (d) are dependent on the correct early event time (of 9) at the end of D and the correct late event time (of 24) at the end of N**

**d1B1:** 'yes' (or clearly implied) **and** mention of 9 **and** 10 **or** 19 **or** a float (oe) of 5 - but not simply 'it will finish before the end of the project' (their answer must contain some form of mathematical argument)

**d2B1:** 'no' (or clearly implied) **and** mention of 9 **and** 17 **or** 26 **or** a delay (oe) of 2 – but not simply 'it will finish late' (their answer must contain some form of mathematical argument)



Question Number	Scheme	Marks
7.	Maximise ( $P =$ ) $40x + 60y + 85z$	B1
	Subject to: $x + y + z \geq 280$	B1
	$\frac{7}{20}(x + y + z) \leq x$ which simplifies to $13x \geq 7y + 7z$	M1 A1
	$\frac{1}{5}(x + y + z) \geq z$ which simplifies to $x + y \geq 4z$	M1 A1
	$\frac{x}{400} + \frac{y}{300} + \frac{z}{200} \leq 1$ which simplifies to $3x + 4y + 6z \leq 1200$	M1 A1
	$(x, y, z \geq 0)$	
		<b>8 marks</b>

**Notes for Question 7**

**1B1:** Expression correct (or  $0.4x + 0.6y + 0.85z$ ) together with 'maximise' or 'max' but not 'maximum' – isw if coefficients are subsequently simplified but either  $40x + 60y + 85z$  or  $0.4x + 0.6y + 0.85z$  must be seen at some point for this mark to be awarded

**2B1:** CAO

**1M1:** Correct method:  $\frac{7}{20}(x + y + z) \bullet x$  where  $\bullet$  is any inequality or  $=$ . The bracket must be present or implied by later working. An exact equivalent answer (with or without integer coefficients but with correct inequality sign) with no working can score M1. Accept equivalent fractions or decimals for  $7/20$  but not  $35\%$  (unless later converted to a correct fraction/decimal)

**1A1:** CAO – answer must have integer coefficients with like terms collected i.e.  $k(13x \geq 7y + 7z)$  for any positive integer  $k$  - the correct answer with no working can score M1 A1

**2M1:** Correct method:  $\frac{1}{5}(x + y + z) \bullet z$  where  $\bullet$  is any inequality or  $=$ . The bracket must be present or implied by later working. An exact equivalent answer (with or without integer coefficients but with correct inequality sign) with no working can score M1. Accept equivalent fractions or decimals for  $1/5$  but not  $20\%$  (unless later converted to a correct fraction/decimal)

**2A1:** CAO – answer must have integer coefficients with like terms collected i.e.  $k(x + y \geq 4z)$  for any positive integer  $k$  - the correct answer with no working can score M1 A1

**3M1:** Correct complete method:  $\frac{x}{400} + \frac{y}{300} + \frac{z}{200} \bullet 1$  (oe) where  $\bullet$  is any inequality or  $=$ . An exact equivalent answer (with or without integer coefficients but with correct inequality sign) with no working can score M1

**3A1:** CAO – answer must have integer coefficients with like terms collected i.e.  $k(3x + 4y + 6z \leq 1200)$  for any positive integer  $k$  - the correct answer with no working can score M1 A1

**Condone  $s, m$  and  $l$  for  $x, y$  and  $z$  for full marks – any other letter used then please send to review (unless clearly defined and then award as per the scheme)**

